

Signatures of Superfluidity in Dilute Fermi Gases near a Feshbach Resonance

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We present a brief account of the most salient properties of vortices in dilute atomic Fermi superfluids near a Feshbach resonance.

PACS numbers: 03.75.Ss

1. INTRODUCTION

Since the creation of the first degenerate Fermi gas by DeMarco and Jin of ^{40}K atoms and the optical trapping of ^6Li atoms by Thomas' group¹ the last year and a half has produced an enormous experimental progress in the study of dilute atomic Fermi gases. The ability to manipulate the strength of the interaction by means of the Feshbach resonance opened extraordinary opportunities both from the experimental and theoretical points of view. There is no doubt in anybody's mind that dilute atomic Fermi clouds near a Feshbach resonance should become superfluid, with a pairing gap of the order of the Fermi energy at sufficiently low temperatures. The challenge from the experimental point of view is to realize these superfluids and especially to demonstrate unambiguously the onset of superfluidity, or at least the formation of a condensate and be able to study its properties. A real breakthrough was the creation and the subsequent study of the expansion of a strongly interacting degenerate Fermi gas². After that experimentalists have been able to study the formation of extremely weakly bound molecules (which we shall often refer to as dimers)³, the decay properties of ensembles of such dimers⁴, the BEC of dimers⁵, a number of features of the BCS to BEC crossover⁶, the collective oscillations⁷, the formation of some kind of condensate, with some still unclear properties⁸ and finally the appearance

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of a gap in the excitation spectrum⁹.

Leggett and others have envisioned theoretically such a BCS to BEC crossover¹⁰ and were able to describe qualitatively its main features. Qualitative features of the BCS dilute atomic Fermi superfluid have been discussed by a number of authors in recent years¹¹. The theoretical description was based essentially on the weak coupling BCS formalism, which is known to over predict the value of the gap by a significant factor¹². The crossover theory of Leggett and its followers was based on a more or less straightforward extension of the weak coupling BCS formalism to the strong coupling regime. In the BEC limit there is an equally significant correction of this results¹³. As it was noted by Bertsch¹⁴, a dilute Fermi system acquires universal properties at, what nowadays we call, the Feshbach resonance. The initial studies of the Bertsch MBX challenge showed that such a system is stable^{15,16}. Only relatively recently that was confirmed both theoretically^{17,18,19} and experimentally².

The discussion of some general properties of these systems, the character of the collective oscillations in trapped dilute atomic Fermi gases near a Feshbach resonance, which we planned to cover as well, along with the discussion of the properties of atom-dimer mixtures, see Refs. 20, are skipped here due to space limitations.

2. Superfluid LDA and the Vortex State

From the theoretical point of view the challenge is to be able to predict and describe in a controllable manner the properties of these systems, which necessitates the development of accurate theoretical tools. We have shown recently how to extend the density functional theory²¹ to superfluid fermion systems, by creating the so called Superfluid LDA (SLDA)^{22,23}. In the case of SLDA one needs to know the dependence of the energy density as a function of both normal and anomalous densities, unlike LDA when only the dependence on the normal density is sufficient. The rather accurate calculations of Refs. 17,18 allows us to construct the energy density functional (EDF) in the case of infinite homogeneous systems. Using the recent results of Chang *et al.* in Ref. 18 we decided to re-parameterize this EDF and extend that parameterization away from the Feshbach resonance. In terms of the single quasi-particle wave functions, which define the normal and anomalous densities the EDF of a superfluid system in the SLDA approach has the form:

$$\mathcal{E}_S(\mathbf{r})n(\mathbf{r}) = \frac{\hbar^2}{m} \left\{ \frac{1}{2} \tau(\mathbf{r})n(\mathbf{r}) + \beta \left[\frac{1}{n(\mathbf{r})a^3} \right] n(\mathbf{r})^{5/3} + \gamma \left[\frac{1}{n(\mathbf{r})a^3} \right] \frac{|\nu(\mathbf{r})|^2}{n(\mathbf{r})^{1/3}} \right\},$$

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$$n(\mathbf{r}) = \sum_{\alpha} |v_{\alpha}(\mathbf{r})|^2, \quad \tau(\mathbf{r}) = \sum_{\alpha} |\nabla v_{\alpha}(\mathbf{r})|^2, \quad \nu(\mathbf{r}) = \sum_{\alpha} v_{\alpha}^*(\mathbf{r}) u_{\alpha}(\mathbf{r}),$$

where the spin degrees of freedom have been suppressed for the sake of simplicity. The dimensionless functions $\beta(x)$ and $\gamma(x)$ can be easily determined using the recent the results of Ref. 18. The SLDA equations have the same formal structure as the Bogoliubov-de Gennes equations for a system with density dependent contact interaction. In spite of the formal resemblance these equations describe such systems exactly, beyond the meanfield approximation, namely the ground state energy and number density distribution.

The local anomalous density $\nu(\mathbf{r})$ is unfortunately strictly speaking a diverging quantity and its evaluation requires a well defined regularization procedure. The evaluation of the pairing field also requires a renormalization procedure and so does actually the evaluation of the total ground state energy. The principles of the regularization and renormalization procedures have been described by us previously, see Refs. 22,23. In the region of the Feshbach resonance, where the pairing gap is of order of the Fermi energy $\varepsilon_F = \hbar^2(3\pi^2n)^{2/3}/2m$ these procedures have to be changed somewhat, in order to ensure a better convergence of various quantities, number density, pairing field, total energy, etc. and these details will be presented elsewhere²⁴. It suffices to add that unlike a number of other methods suggested recently in literature, aimed at dealing with similar divergences, see Refs. 25 and discussion in Ref. 26, and which require an active Hilbert space of a size $10^3 \dots 10^5 N$, in our approach the size of the active Hilbert space is typically much less than $10N$, where N is the total number of particles.

The form of the EDF presented above is not unique to a certain extent. In particular one could have considered a density dependent mass in the kinetic energy term, as is often done in nuclear physics. Until full many-body calculations of the homogeneous matter of the kind described in Refs. 17,18,19 will provide more detailed information about the properties of such systems, beyond the energy per particle and the pairing gap as a function of the parameter $1/na^3$, there is no unambiguous way one can determine whether the effective mass is different or not from the bare mass. There is no spin-orbit coupling as well and so far there is no compelling argument to expect its presence. However, one can expect gradient terms, in particular a dependence of the EDF on $\nabla n(\mathbf{r})$. In principle such terms could be evaluated, but their role is not expected to be ever dominant, though it could be significant. Phenomenologically, in nuclear physics it is established that such terms are quite important. The reason they are important in nuclear physics is because the radius of the interaction is comparable with the Fermi wave length. This is definitely not the case of dilute atomic gases for which $nr_0^3 \ll 1$ always (here r_0 is of the order of at most 100 Å or so, the so

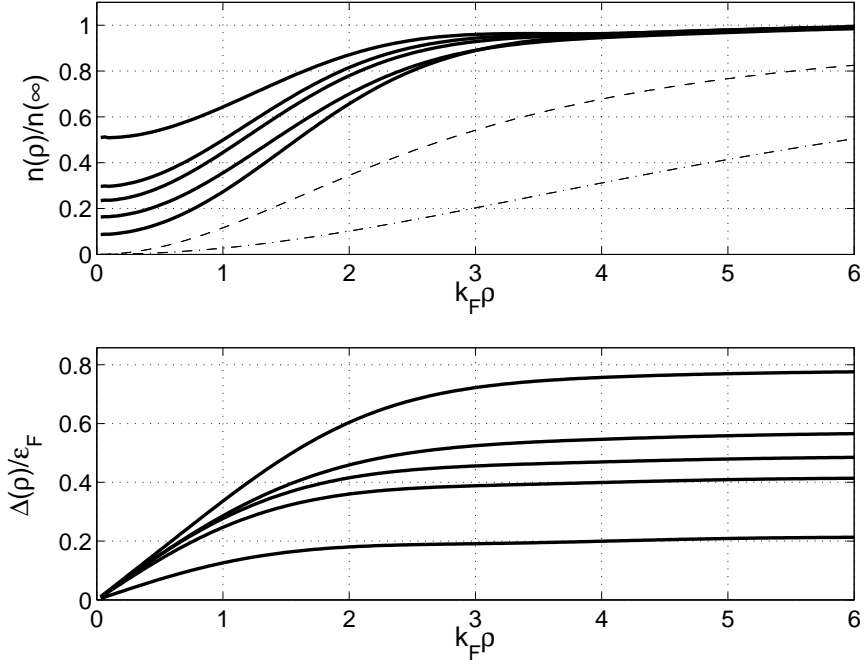


Fig. 1. The upper panel shows the number density profile around a vortex core, while the lower panel shows the profile of the pairing field. In the upper panel various curves correspond to $1/k_F a = (-0.5, -0.1, 0, 0.1, 0.3)$, from the highest to the lowest respectively. The order is reversed in the lower panel. We show also the (approximate) density profile of a vortex in a Bose dilute atomic gas of the same number density, with a dashed line for the case $na^3 = 10^{-3}$ and with a dot-dashed line for the case $na^3 = 10^{-5}$.

called van der Waals length). One can come up with a similar qualitative argument in favor of an effective mass close in value to the bare mass, which was implicitly assumed by us. It is worth noting also that the present EDF is quite distinct from others suggested recently in literature, see for example Ref. 27, which are typically based on one or another incarnation of the crossover model due to Leggett¹⁰.

At this point all the elements needed, in order to perform a full self-consistent calculation of the vortex properties in a dilute atomic Fermi gas near a Feshbach resonance, are known. Without dwelling into technical details^{23,24} we shall briefly discuss the most salient features. In Fig. 1 we show the number density profile $n(\rho)/n(\infty)$ and the profile of the pairing field $\Delta(\rho)/\epsilon_F$ around a vortex core, where $\rho = (x, y)$ and the z -axis is along the vortex core. At the Feshbach resonance the asymptotic value of the pairing field is approximately one half the free Fermi energy of the free gas.

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The actual Fermi energy at the Feshbach resonance is $\approx 0.8\varepsilon_F$. As one can see from the lower panel in Fig. 1 the pairing field has a rather dramatic dependence on the scattering length¹⁸.

What came as a big surprise before²³ and it is confirmed by the present results, based on a more accurate EDF, is the unexpected appearance of the prominent vortex core number density depletion shown in the upper panel of Fig. 1. For comparison we show there as well the number density profile of two vortices in dilute Bose gases of the same density and two different values of the corresponding atomic scattering length. As one can see the size of the vortex core is only 2-4 times smaller in the Fermi case than the vortex core in the BEC case. This fact let us conclude that a direct visualization of vortices should be easily achievable in the case of atomic Fermi superfluids around the Feshbach resonance.

3. Conclusions

The onset of superfluidity should undoubtedly be demonstrated by exciting and putting in evidence a superflow, and vortices are just about the only such modes in which superflow can be seen in such systems. Moreover, these vortices are also expected to form an Abrikosov lattice. We presented an analysis, based on a newly developed extension of LDA to superfluid systems, Superfluid LDA (SLDA), of the properties of vortices at and near a Feshbach resonance. Surprisingly, like in the case of Bose superfluids, vortices in Fermi superfluids near a Feshbach resonance share common features, which should make them easily detectable. Vortices develop a pronounced density depletion in the core of a size comparable to the size of a core in a dilute atomic Bose gas.

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